



Sampling interval and estimated betas: Implications for the presence of transitory components in stock prices[☆]

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ABSTRACT

We provide a theoretical framework to explain the empirical finding that the estimated betas are sensitive to the sampling interval even when using continuously compounded returns. We suppose that stock prices have both permanent and transitory components. The discrete time representation of the beta depends on the sampling interval and two components labeled “permanent and transitory betas”. We show that if no transitory component is present in stock prices then no sampling interval effect occurs. However, the presence of a transitory component implies that the beta is an increasing (decreasing) function of the sampling interval for more (less) risky assets. In our framework, assets are labeled risky if their “permanent beta” is greater than their “transitory beta” and vice versa for less risky assets. Simulations show that our theoretical results provide good approximations for the estimated betas in small samples. We provide empirical evidence about the presence of negative serial correlation and mean reversion in the returns of the portfolios considered. We discuss why our model is better able to provide an explanation for this sampling interval effect than other models in the literature.

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1. Introduction

The Capital Asset Pricing Model (CAPM) has been the object of numerous studies over the past thirty years. Developed by Sharpe (1964) and Lintner (1965), it is based on the assumption that investors are risk-averse and construct their portfolios according to a mean–variance criterion. The basic relation says that in equilibrium there exists a linear relation between the return of a given asset or portfolio and the return of the market portfolio.

An empirical feature that has attracted some attention is the fact that the estimated beta (or systematic risk of an asset or portfolio) is sensitive to the sampling interval used to compute the returns. This “interval effect” has been analyzed in relation with another anomaly, the size effect, which shows a significant relation between returns and the market values of firms. Banz (1981) has analyzed this effect and showed that the smaller a firm is the higher is its expected return. For the interval effect, the

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empirical studies show that changes in the sampling interval used induce a bias in the estimate of the systematic risk whose magnitude depends on the size of the firms as measured by their market value.

According to Pogue and Solnik (1974), Roll (1981) and Reinganum (1982), the beta is underestimated for small firms and overestimated for large firms when using daily data. Such a bias is attributed to the small frequency at which the assets of small firms are transacted (Scholes and Williams, 1977; Dimson, 1979) and more generally to frictions in the exchange process (Cohen et al., 1983). According to Cohen et al., prices adjust following the arrival of information and the adjustment delays are related to the size of firms. Accordingly, for large firms with greater trading volume, the adjustment delays are shorter than for small firms whose trading volume is smaller. The infrequent exchange for small firms is accompanied with the non-synchronization of individual prices in relation to the market index which induces an intertemporal correlation between returns and an autocorrelation in the market returns. In this study, we shall not be concerned about such relations holding at very short sampling intervals where market microstructure effects are operative. Rather, we shall concentrate on ranges of sampling intervals where these market microstructure effects are not present; for example constructing returns from monthly to annual intervals. It is possible that intraday periodicity or seasonality may have an impact on returns at longer horizons, a full investigation of which is outside the scope of this paper. However, we believe that such effects are unlikely. For instance, Andersen and Bollerslev (1997) document a strong intradaily periodic pattern in the volatility. However, their model used to describe this feature implies a reduction in the overall level of the interdaily return autocorrelations (pp. 127–128). To our knowledge, there is no evidence of spillover from high frequency intradaily features to temporal dependence in returns at horizons between one month and a year, which are considered in this paper.

To be precise about the terminology, we use the following definitions. First, $P_i(th)$ denotes the dividend-reinvested price index for security i measured over a sampling interval of h periods. The relative prices $P_i(th)/P_i((t-1)h)$ are then the h -period returns. For a given portfolio with, say, N securities the buy-and-hold returns are given by $N^{-1} \sum_i [P_i(th)/P_i((t-1)h)]$ and the continuously compounded returns by $\log(N^{-1} \sum_i P_i(th)/N^{-1} \sum_i P_i((t-1)h))$.

On a theoretical level, Levhari and Levy (1977) and Hawawini (1980) show a relation between the beta and the sampling interval in the case where the returns are computed using the relative prices $P(th)/P((t-1)h)$ to define the h -period returns, i.e. the buy-and-hold returns. In such cases, the “interval effect” is simply due to an accounting issue. Handa et al. (1989, 1993) show clearly that an interval effect is present empirically and that the betas of more risky assets increase as the sampling interval increases, while the betas of less risky assets are decreasing. Their results also show that the estimated betas approach that of the market portfolio (i.e. one) when the sampling interval gets smaller. They argue that if continuously compounded returns are used, no such interval effect should hold if markets are efficient.

An interesting fact is that a similar sampling interval effect is present empirically when using continuously compounded returns. This was shown as early as in the study by Smith (1978) and also in the more specialized analyses of Corhay (1990) and Defrère (1995). To further corroborate this empirical fact, we provide an empirical study along the line of Handa et al. (1989) and show that the sampling interval effect is very similar whether using buy and hold or continuously compounded returns.

The purpose of our study is then to provide a theoretical framework where interval effects are present even when using continuously compounded returns. We suppose that stock prices have both permanent and transitory components. The permanent component is a standard geometric Brownian motion with constant volatility while the transitory component is a stationary Ornstein–Uhlenbeck process. We derive the discrete time representation of the beta which depends on the sampling interval and two components labeled “permanent and transitory betas” (to be defined explicitly). We show that if no transitory component is present in stock prices then no sampling interval effect occurs. However, the presence of a transitory component implies that the beta is an increasing (decreasing) function of the sampling interval for more risky (less risky) assets. In our framework, assets are labeled risky if their “permanent beta” is greater than their “transitory beta” and vice versa for less risky assets. Simulations show that our theoretical results provide good approximations for the estimated betas in small samples.

According to our results, the presence of a transitory component is the crucial element to explain the “interval effect”, without it no such effect should be present. This transitory component is similar to that used by Fama and French (1988) and Poterba and Summers (1988) to explain the presence of negative serial correlation in returns at long horizons. Our theoretical results and the presence of the interval effect empirically found can be perceived as indirect evidence for the presence of a transitory component in stock prices. We nevertheless provide empirical evidence to that effect. First, we consider estimates of simple ARMA(1,1) processes for the various portfolios when estimated using 6 or 12 months sampling intervals. These show evidence of negative serial correlation in the portfolios' returns. Second, we evaluate long-horizon regressions of the type considered by Fama and French (1988) based on 1 to 10 year returns and show that when tested jointly the parameter estimates support the presence of mean reversion.

We discuss the implication of our results in relation to the previous literature, in particular the work of Lo and MacKinlay (1990). Given the evidence presented, we believe that our simple model with transitory components is better able to explain the full pattern of the estimates of the betas using various sampling intervals. As evidenced by the literature that followed the work of Lo and MacKinlay (1990) it is also consistent with the lead-lag pattern and, moreover, able to explain the pattern of the estimates of the betas for both small and large firms, while the framework of Lo and MacKinlay (1990) can only provide an explanation for the decrease in the estimated betas for small firms.

The paper is structured as follows. Section 2 presents the empirical results showing the presence of a sampling interval effect for estimated betas continuously compounded as well as buy and hold returns. Section 3 defines the basic model in continuous time and derives its discrete time representation. In Section 4, we discuss the properties of the beta in relation to the sampling interval and the limiting behavior of its estimate. Section 5 provides simulation evidence that supports the theoretical results. Section 6 presents empirical evidence that documents the presence of transitory components and mean reversion in the various

portfolios considered. Section 7 provides a discussion of our results in relation to the previous literature. Section 8 offers concluding comments. Technical derivations and details on various computations are contained in an appendix.

2. The empirical facts

In this section, we document the presence of the sampling interval effect on estimated betas using continuously compounded returns. The setup is basically the same as that used in Handa et al. (1989) with an extended sample. Hence, we provide only a brief summary of the procedure and refer the reader to that paper for more information.

We use all stocks listed on the Center for Research in Security Prices (CRSP) monthly tape. This includes all the New York Stock Exchange (NYSE) firms for the period 1926:1 to 2008:12 and the American Stock Exchange (AMEX) securities for the period 1964:1 to 2008:12. We rank all securities according to their beginning-of-year equity market values and divide them into 20 equal portfolios (with the portfolio labeled 1 containing the smallest 5% firms and the portfolio labeled 20 containing the largest 5% firms). This ranking and grouping is repeated every year.

We consider estimating market-model betas for six sampling intervals: 1, 2, 3, 4, 6 and 12 months. For all intervals, we use the equal-weighted sample mean returns as the market portfolio proxy. The betas are estimated using a 15 year overlapping window.

Table 1

Sample means of market-value portfolio betas and standard errors for different return measurement intervals (NYSE and AMEX, 1926:1–2008:12).

Sample means: Buy-and-hold returns							Sample means: Continuously compounded returns						
Portfolio	Intervals (months)						Portfolio	Intervals (months)					
	12	6	4	3	2	1		12	6	4	3	2	1
MV1	1.74	1.68	1.62	1.57	1.53	1.38	MV1	1.46	1.45	1.45	1.41	1.38	1.30
MV2	1.49	1.43	1.43	1.38	1.31	1.26	MV2	1.35	1.32	1.34	1.29	1.25	1.22
MV3	1.27	1.28	1.29	1.27	1.22	1.18	MV3	1.23	1.24	1.25	1.23	1.19	1.17
MV4	1.25	1.24	1.23	1.21	1.18	1.15	MV4	1.22	1.20	1.21	1.17	1.16	1.14
MV5	1.16	1.19	1.19	1.15	1.14	1.11	MV5	1.17	1.17	1.17	1.13	1.13	1.10
MV6	1.16	1.13	1.11	1.10	1.09	1.08	MV6	1.14	1.11	1.10	1.08	1.08	1.08
MV7	1.09	1.08	1.07	1.05	1.06	1.05	MV7	1.09	1.07	1.06	1.05	1.06	1.04
MV8	1.06	1.07	1.07	1.06	1.05	1.04	MV8	1.09	1.08	1.07	1.06	1.06	1.04
MV9	1.00	1.00	1.01	1.02	1.02	1.03	MV9	1.01	1.01	1.01	1.02	1.03	1.03
MV10	0.98	1.00	0.99	0.99	1.00	1.01	MV10	1.01	1.03	1.01	1.01	1.02	1.02
MV11	0.96	0.97	0.96	0.97	0.99	0.99	MV11	0.98	0.99	0.97	0.98	1.00	1.00
MV12	0.95	0.95	0.94	0.95	0.97	0.98	MV12	0.96	0.97	0.96	0.97	0.98	0.99
MV13	0.87	0.87	0.89	0.90	0.91	0.94	MV13	0.90	0.90	0.91	0.92	0.93	0.95
MV14	0.88	0.88	0.88	0.91	0.92	0.94	MV14	0.90	0.90	0.90	0.93	0.93	0.95
MV15	0.80	0.82	0.84	0.85	0.87	0.89	MV15	0.83	0.85	0.86	0.87	0.89	0.90
MV16	0.76	0.77	0.79	0.82	0.83	0.87	MV16	0.79	0.80	0.82	0.85	0.85	0.88
MV17	0.75	0.76	0.77	0.79	0.82	0.85	MV17	0.78	0.78	0.79	0.82	0.83	0.86
MV18	0.68	0.69	0.71	0.74	0.77	0.82	MV18	0.72	0.73	0.74	0.77	0.79	0.83
MV19	0.62	0.64	0.65	0.69	0.71	0.75	MV19	0.66	0.67	0.68	0.72	0.72	0.76
MV20	0.52	0.55	0.56	0.59	0.61	0.68	MV20	0.56	0.59	0.59	0.63	0.63	0.69
Standard errors: Buy-and-hold returns							Standard errors: Continuously compounded returns						
Portfolio	Intervals (months)						Portfolio	Intervals (months)					
	12	6	4	3	2	1		12	6	4	3	2	1
MV1	0.304	0.256	0.177	0.194	0.268	0.177	MV1	0.186	0.156	0.099	0.121	0.160	0.120
MV2	0.178	0.147	0.164	0.157	0.158	0.132	MV2	0.115	0.100	0.094	0.100	0.109	0.094
MV3	0.159	0.113	0.116	0.115	0.116	0.104	MV3	0.103	0.071	0.090	0.085	0.092	0.089
MV4	0.113	0.104	0.080	0.098	0.075	0.077	MV4	0.103	0.082	0.073	0.074	0.067	0.068
MV5	0.111	0.080	0.108	0.080	0.078	0.078	MV5	0.080	0.071	0.085	0.070	0.071	0.072
MV6	0.049	0.053	0.067	0.069	0.076	0.062	MV6	0.040	0.039	0.063	0.060	0.063	0.059
MV7	0.063	0.051	0.047	0.050	0.050	0.058	MV7	0.052	0.036	0.045	0.043	0.049	0.054
MV8	0.054	0.037	0.031	0.039	0.033	0.035	MV8	0.037	0.029	0.026	0.032	0.031	0.036
MV9	0.084	0.077	0.060	0.062	0.050	0.046	MV9	0.079	0.066	0.054	0.056	0.049	0.046
MV10	0.071	0.085	0.082	0.067	0.070	0.056	MV10	0.068	0.077	0.070	0.056	0.059	0.051
MV11	0.046	0.055	0.055	0.041	0.040	0.033	MV11	0.042	0.043	0.044	0.029	0.036	0.029
MV12	0.063	0.037	0.035	0.030	0.032	0.022	MV12	0.057	0.037	0.031	0.028	0.026	0.020
MV13	0.068	0.043	0.049	0.064	0.060	0.047	MV13	0.053	0.042	0.044	0.054	0.052	0.043
MV14	0.071	0.057	0.065	0.061	0.067	0.060	MV14	0.052	0.043	0.058	0.057	0.066	0.058
MV15	0.077	0.063	0.066	0.079	0.085	0.072	MV15	0.060	0.052	0.055	0.064	0.073	0.063
MV16	0.089	0.070	0.069	0.068	0.077	0.065	MV16	0.072	0.051	0.056	0.052	0.064	0.055
MV17	0.096	0.053	0.068	0.066	0.069	0.072	MV17	0.083	0.042	0.068	0.061	0.066	0.067
MV18	0.098	0.065	0.090	0.082	0.081	0.092	MV18	0.088	0.043	0.088	0.073	0.078	0.086
MV19	0.101	0.073	0.084	0.075	0.079	0.089	MV19	0.088	0.066	0.075	0.067	0.073	0.081
MV20	0.100	0.077	0.086	0.086	0.085	0.096	MV20	0.098	0.077	0.080	0.075	0.078	0.088

What are reported in Table 1 are time series averages of estimated betas for the various portfolios along with their standard errors. To properly compare the sampling interval effect, we report results for both cases where the estimates are constructed using buy-and-hold returns (simple returns adjusted for dividends over the relevant interval) and their continuously compounded counterpart (the logarithm of these returns).

The results shown for the buy-and-hold returns basically confirm the results in Handa et al. (1989). A sampling interval effect is clearly present especially for the extreme portfolios. For portfolio 1 (the smallest firms) the estimated betas increase as the sampling interval increases and for portfolio 20 (the largest firms) they decrease. As discussed in Handa et al. (1989), this effect is expected from a simple accounting issue. However, such an accounting issue is not present with continuously compounded returns and, in this case, no sampling interval effect should be present if markets are efficient. However, the results in Table 1 clearly show that the same sampling interval effect remains with log-returns. For many portfolios, it is even more pronounced.

These results suggest that more than a simple accounting issue is responsible for the explanation of the sampling interval effect on estimated betas. The rest of this paper intends to provide an analytical framework to assess potential sources for this empirical fact.

3. The basic model

We denote by $P_1(t)$, the price of a stock or a portfolio at date t , and by $P_2(t)$ the price of the market portfolio at the same date. We suppose that each price has two multiplicative components. One, denoted by $P_i^a(t)$ represents the transitory component while the other, denoted by $P_i^b(t)$, is the permanent component. Hence, we have:

$$P_i(t) = P_i^a(t)P_i^b(t) \quad (i = 1, 2).$$

The assumption of a permanent and transitory component for stock prices is frequently made (see, e.g., Poterba and Summers (1988), and Fama and French (1988)). It is usually motivated by the fact that it allows negative correlation in returns over long horizons which has been shown to be present empirically. Also, we denote by lower cases, the logarithm of the respective components, i.e.:

$$p_i^j(t) = \ln P_i^j(t) \quad (i = 1, 2; \quad j = a, b).$$

The continuous time model describing the time paths of each component is intentionally kept simple to highlight the features of interest and is not intended as a precise description of the behavior of stock prices at all sampling intervals. It is intended to be a useful approximation for the sampling intervals of interest, namely monthly to yearly, for which positive serial correlation due to market microstructure effects do not hold but for which negative serial correlation is a possibility in the presence of transitory components. Accordingly, the transitory $P_i^a(t)$ and permanent $P_i^b(t)$ components are governed by the following stochastic differential equations, defined over $[0, N]$, with N the span of the data:

$$\begin{aligned} dp_i^a(t) &= -\gamma_i p_i^a(t)dt + \sigma_i^a dW_i^a(t), \\ dp_i^b(t) &= \alpha_i p_i^b(t)dt + \sigma_i^b P_i^b(t)dW_i^b(t), \end{aligned} \tag{1}$$

for $i=1, 2$, with $W^j(t) = (W_1^j(t), W_2^j(t))'$, ($j = a, b$) where $W^a(t)$ and $W^b(t)$ are independent standard Wiener processes, i.e., continuous time zero mean Gaussian processes defined on $[0, 1]$ with covariance function $E[W^j(t)W^j(s)] = \min(t, s)$ for $j = a, b$. We make the following assumptions:

$$\begin{aligned} E[W_1^j(t)W_2^j(t)] &= \rho_j, \quad 0 \leq |\rho_j| \leq 1, \quad j = a, b; \\ \gamma_i > 0, \alpha_i > 0, \quad \sigma_i^a > 0, \sigma_i^b > 0, \quad i = 1, 2. \end{aligned}$$

The stochastic differential equation describing the dynamics of the transitory component specifies that the logarithm of the transitory component of prices $P_i^a(t)$ is an Ornstein–Uhlenbeck process. Accordingly, the long-term effect of a shock on the level of the transitory component is zero and constraining the parameters γ_i ($i = 1, 2$) to be positive ensures mean reversion. On the other hand, the dynamics of the permanent component $P_i^b(t)$ is governed by a geometric Brownian motion. The parameter α_i here reflects mean returns. The parameters $(\sigma_i^j)^2$ ($i = 1, 2$ and $j = a, b$) represent the variances of the noise components $W_i^j(t)$ and are often called diffusion components. The parameter ρ_j accounts for the correlation between the noise of the temporary components ($j = a$) or permanent components ($j = b$) of the price of the asset (or portfolio) and the price of the market portfolio. Such specifications are often encountered in the finance literature. For example, a geometric Brownian motion is often postulated for risky stock prices while an Ornstein–Uhlenbeck is used for riskless assets (e.g., the short-term interest rate on a safe asset); see, e.g., Merton (1973) and Black and Scholes (1973).

The assumption of the independence of the Wiener processes $W^a(t)$ and $W^b(t)$, allows us to write the system (1) as two sub-systems; namely

$$dp_i^a(t) = -\gamma_i p_i^a(t)dt + \sigma_i^a dW_i^a(t), \tag{2}$$

and

$$dp_i^b(t) = \left(\alpha_i - \frac{(\sigma_i^b)^2}{2} \right) dt + \sigma_i^b dW_i^b(t), \quad (3)$$

for $i = 1, 2$. The systems (2) and (3) have the following solutions:

$$p_i^a(t) = p_i^a(0) \exp(-\gamma_i t) + \sigma_i^a \int_0^t \exp(-\gamma_i(t-s)) dW_i^a(s), \quad (4)$$

and

$$p_i^b(t) = p_i^b(0) + \left(\alpha_i - \frac{(\sigma_i^b)^2}{2} \right) t + \sigma_i^b W_i^b(t), \quad (5)$$

for $i = 1, 2$ (see, e.g. Theorems 8.2.2 and 8.4.3 of [Arnold \(1974\)](#)). These solutions show that the logarithm of the transitory component is stationary while the logarithm of the permanent component is an integrated component with a linear trend. For simplicity and without loss of generality, we suppose, in what follows, that $p_i^j(0) = 0$ ($i = 1, 2; j = a, b$).

To compare our model with that of [Poterba and Summers \(1988\)](#) and [Fama and French \(1988\)](#), we need to derive the discrete time representation. To that effect, we define the sampling interval h such that $Th = N$ with T the number of observations and N the span of the data. We have the following discrete time solutions for Eqs. (4) and (5).

Proposition 1. Let $p_i^a(t)$ and $p_i^b(t)$ ($i = 1, 2$) be defined by Eq. (1), then the discrete time solutions for a sampling interval h are given by:

$$p_i^a(th) = \exp(-\gamma_i h) p_i^a((t-1)h) + u_i(th), \quad (6)$$

and

$$p_i^b(th) - p_i^b((t-1)h) = \left(\alpha_i - \frac{(\sigma_i^b)^2}{2} \right) h + v_i(th), \quad (7)$$

for $i = 1, 2$ and $t = 1, \dots, T$; with

$$u_i(th) = \sigma_i^a \int_{(t-1)h}^{th} \exp(-\gamma_i(th-s)) dW_i^a(s),$$

and

$$v_i(th) = \sigma_i^b (W_i^b(th) - W_i^b((t-1)h)).$$

The proof follows immediately from the solutions (4) and (5). The errors $u_i(th)$ and $v_i(th)$ have mean zero, are independent and are identically normally distributed. The moments of order two satisfy (for $i = 1, 2$):

$$\begin{aligned} E[u_i^2(th)] &= \frac{1 - \exp(-2\gamma_i h)}{2\gamma_i} (\sigma_i^a)^2, \\ E[u_1(th)u_2(th)] &= \frac{1 - \exp(-(\gamma_1 + \gamma_2)h)}{(\gamma_1 + \gamma_2)} \rho_a \sigma_1^a \sigma_2^a, \\ E[v_i^2(th)] &= (\sigma_i^b)^2 h, \\ E[v_1(th)v_2(th)] &= \rho_b \sigma_1^b \sigma_2^b h. \end{aligned}$$

We now need to define the returns over a horizon of h periods. Supposing that dividends are zero (or that they are reinvested), the instantaneous returns are $R(t) = dP(t)/P(t)$. Given the discrete time solutions of prices, the discrete time solution for returns over h periods is defined by:

$$R_i(th) = (1 - L^h) (p_i^a(th) + p_i^b(th)), \quad (8)$$

where L^h is the lag operator such that $Lx_t = x_{t-h}$. A representation for $R_i(th)$ in terms of the errors $v_i(th)$ and $u_i(th)$ is given by:

$$R_i(th) = \left(\alpha_i - \frac{(\sigma_i^b)^2}{2} \right) h + v_i(th) + (1 - \exp(-\gamma_i h) L)^{-1} (1 - L) u_i(th). \quad (9)$$

Using the notation $R_{th} = (R_1(th), R_2(th))'$, we can write

$$R_{th} = \Psi h + \eta_{th},$$

where $\Psi = (\Psi_1, \Psi_2)'$, $\eta_{th} = (\eta_1(th), \eta_2(th))'$, $\Psi_i = \alpha_i - (\sigma_i^b)^2/2$ and

$$\eta_i(th) = v_i(th) + (1 - \exp(-\gamma_i h))^{-1} (1 - L) u_i(th).$$

We can use these specifications to derive the following result pertaining to the discrete time representation of returns.

Proposition 2. In discrete time, the returns $R_i(th)$ are characterized by an ARMA(1,1) process with first-order covariance coefficient given by:

$$\text{cov}(R_i(th), R_i((t-1)h)) = -\frac{(1 - \exp(-\gamma_i h))^2}{2\gamma_i} (\sigma_i^a)^2, \quad i = 1, 2. \tag{10}$$

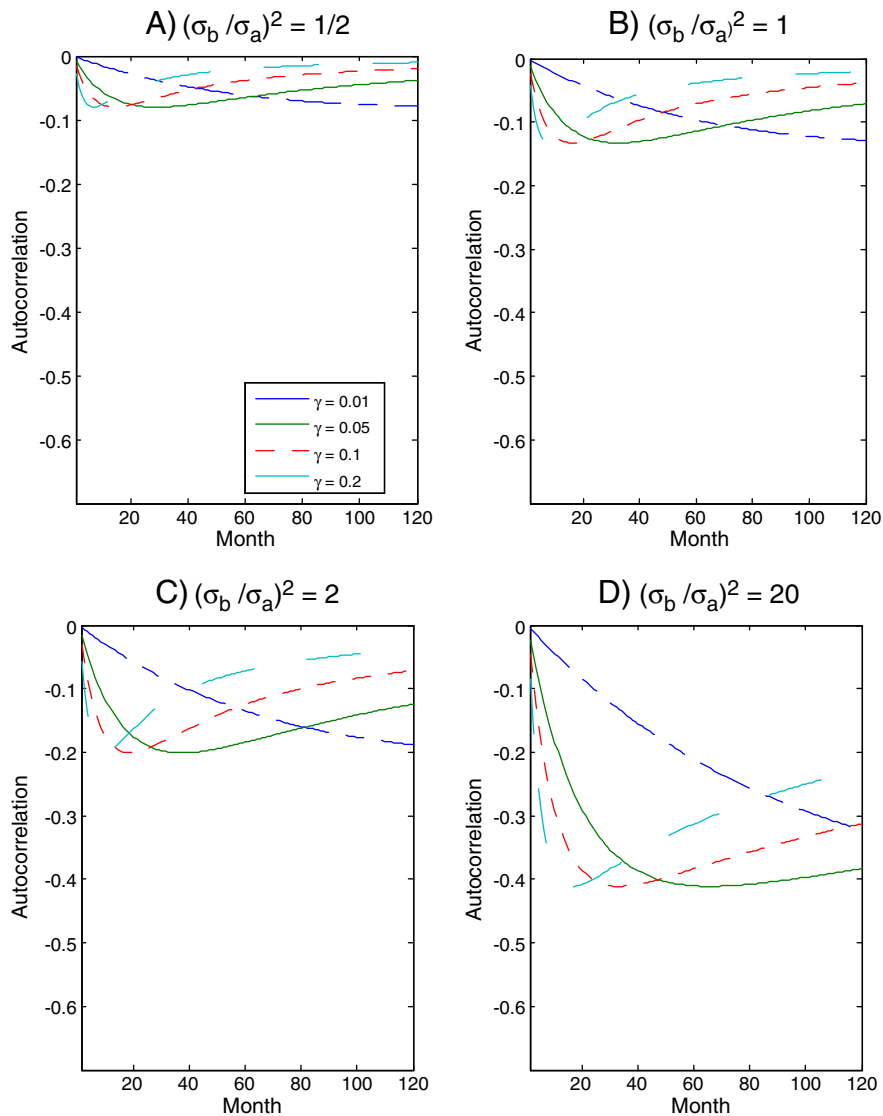


Fig. 1. Autocorrelation functions of stock returns with various parameters.

In particular, for small γ_i or h we have the approximation:

$$\text{cov}(R_i(th), R_i((t-1)h)) \approx -\frac{\gamma_i h^2}{2} (\sigma_i^a)^2, \quad i = 1, 2, \quad (11)$$

so that when either γ_i or h approaches 0, the returns $R_i(th)$ are i.i.d.. For the autocorrelations, we have

$$\text{corr}(R_i(th), R_i((t-1)h)) = -\frac{(1 - \exp(-\gamma_i h))^2 \left(\frac{\sigma_i^a}{\sigma_i^b}\right)^2}{1 + \frac{1 - \exp(-\gamma_i h)}{\gamma_i h} \left(\frac{\sigma_i^a}{\sigma_i^b}\right)^2}, \quad i = 1, 2. \quad (12)$$

and, for small γ_i or h we have the following approximation

$$\text{corr}(R_i(th), R_i((t-1)h)) \approx -\frac{\gamma_i h \left(\frac{\sigma_i^a}{\sigma_i^b}\right)^2}{2 \left[1 + \left(\frac{\sigma_i^a}{\sigma_i^b}\right)^2\right]}, \quad i = 1, 2,$$

so that when either γ_i or h approaches 0, the returns $R_i(th)$ are i.i.d.

This proposition shows that stock returns have the negative autocorrelation that is a function of structural parameters γ_i , h and $(\sigma_i^a/\sigma_i^b)^2$. Fig. 1 presents the plots of the autocorrelation functions over the monthly horizons $h \in [1, 72]$. First, the mean-reversion parameter γ_i determines the shape of the function over h . When γ_i is set to 0.01, the autocorrelation is monotonically decreasing in h over a 6 year period. As γ_i increases to 0.1, it becomes a convex function that exhibits its largest negative autocorrelation over the 2–5 year interval. These values of γ_i are likely to describe the data well as Fama and French (1988) document an empirical evidence that stock returns for the market and decile portfolios have the negative autocorrelation with the U-shaped pattern around 3–5 year periods. When γ_i is set to 0.2, the autocorrelation reaches its lowest level too early before the one-year horizon.

The ratio of the variances of transitory and permanent components, $(\sigma_i^a/\sigma_i^b)^2$, determines the extent to which stock returns are autocorrelated. The top two graphs in Fig. 1 are drawn with the ratio being half and one. For $\gamma_i = 0.05$ or 0.1, the autocorrelation has values of about -0.07 ($(\sigma_i^b/\sigma_i^a)^2 = 1/2$) or -0.12 ($(\sigma_i^b/\sigma_i^a)^2 = 1$) around a 1 year horizon. These values are consistent with the empirical evidence documented by Fama and French (1988). However, their largest negative autocorrelation values are at most -0.08 and -0.13 , respectively. Fama and French (1988) report that for the NYSE market portfolio returns, the autocorrelation has the largest negative value around -0.20 with the sample period from 1941 to 1985 or around -0.40 with the period from 1926 to 1985. Setting higher values of the ratio of the variances, we can obtain minimum autocorrelation values that are empirically consistent. When the ratio is set to 2, the minimum autocorrelation is -0.20 . However, the ratio has to be set to 20 in order to achieve -0.40 for the minimum autocorrelation that corresponds to the estimates obtained in the sample from 1926 to 1985. Nevertheless, these figures in general show that our model with appropriate parameter values satisfies the qualitative properties suggested by the empirical findings in Fama and French (1988).

In summary, the proposition shows that our model satisfies the same qualitative properties as that of Poterba and Summers (1988). In particular, it implies negative correlation in returns that become stronger as the horizon h increases but that this correlation is negligible for short horizons. Also, when the transitory component is null ($\sigma_i^a = 0$ or $\gamma_i = 0$) this correlation disappears and the returns are i.i.d..

In this study, we wish to consider the behavior of the estimator of the systematic risk (the betas) when the sampling interval is allowed to vary. To that effect, we shall adopt different asymptotic frameworks whereby either h decreases to 0 keeping the span N fixed, or keeping h fixed and letting the span N increases.

4. Estimates of beta: Asymptotic properties and implications

We start by defining the notion of the systematic risk beta implied by the model and its limit value as the sampling interval increases or decreases. After a discussion of the population value, we turn to the characterization of the estimates.

4.1. Population values of betas

Definition 1. Let $R_{th} = (R_1(th), R_2(th))'$ be defined by Eq. (9). For a sampling interval h , the systematic risk is defined by:

$$\beta_{0h} = \frac{\text{cov}(R_1(th), R_2(th))}{\text{var}(R_2(th))}. \quad (13)$$

In particular, if $h \rightarrow 0$, we use the notation $\beta_0 = \lim_{h \rightarrow 0} \beta_{0h}$ and if $h \rightarrow \infty$, we use $\tilde{\beta}_{0b} = \lim_{h \rightarrow \infty} \beta_{0h}$.

We have the following representation of β_{0h} as a function of the sampling interval h and the parameters of the model.

Proposition 3. Let $R_{th} = (R_1(th), R_2(th))'$ be defined by Eq. (9) and β_{0h} by Eq. (13). We have:

$$\beta_{0h} = \frac{\rho_b \sigma_1^b \sigma_2^b + \rho_a \sigma_1^a \sigma_2^a \frac{2 - \exp(-\gamma_1 h) - \exp(-\gamma_2 h)}{(\gamma_1 + \gamma_2)h}}{(\sigma_2^b)^2 + (\sigma_2^a)^2 \frac{1 - \exp(-\gamma_2 h)}{\gamma_2 h}}. \tag{14}$$

If $h \rightarrow 0$, we have:

$$\beta_0 = \frac{\rho_b \sigma_1^b \sigma_2^b + \rho_a \sigma_1^a \sigma_2^a}{(\sigma_2^b)^2 + (\sigma_2^a)^2}, \tag{15}$$

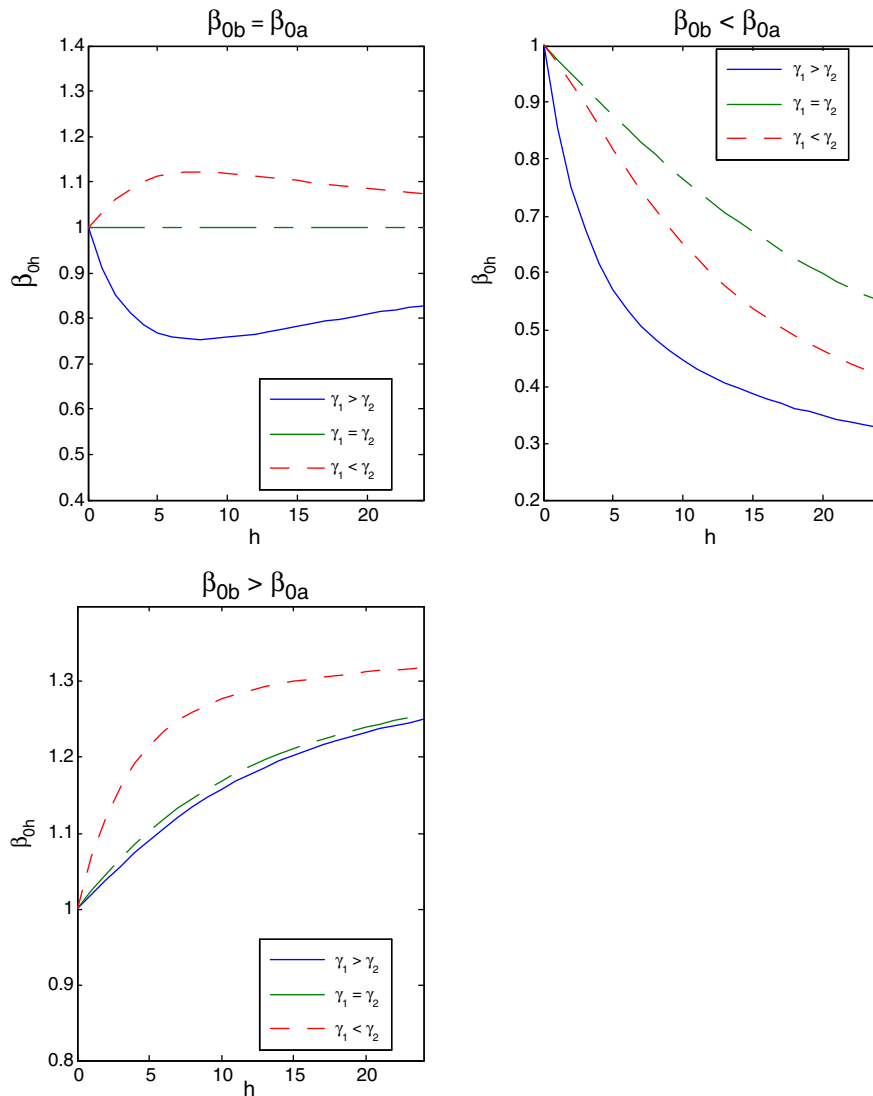


Fig. 2. True values of β_{0h} as a function of h .

and if $h \rightarrow \infty$,

$$\tilde{\beta}_{0b} = \frac{\rho_b \sigma_1^b}{\sigma_2^b}. \quad (16)$$

The expression (14) differs from those of [Levhari and Levy \(1977\)](#) and [Hawawini \(1980\)](#) who present the ratio of an asset's beta computed over h periods relative to that over one period as a function of h and intertemporal cross correlations. The relation (14) suggests that if the transitory component is not present in the assets' prices ($\sigma_1^a = \sigma_2^a = 0$), the true value of beta is independent of the sampling interval and coincides with $\tilde{\beta}_{0b}$ defined as the limit of β_{0h} when h increases. Hence, without the transitory component the sampling interval does not affect the value of the beta. For that reason, we shall refer to $\tilde{\beta}_{0b}$ as the "permanent-beta".

By analogy, we refer to $\tilde{\beta}_{0a} = \rho_a \sigma_1^a / \sigma_2^a$ as the "transitory-beta". It is the value that the beta would take in the absence of a permanent component when h is small. Using this notation, we see that the true value of beta, β_{0h} , is a function of $\tilde{\beta}_{0b}$, $\tilde{\beta}_{0a}$ and h given by:

$$\beta_{0h} = \frac{\tilde{\beta}_{0b} + \tilde{\beta}_{0a} \left(\frac{\sigma_2^a}{\sigma_2^b} \right)^2 \frac{2 - \exp(-\gamma_1 h) - \exp(-\gamma_2 h)}{(\gamma_1 + \gamma_2)h}}{1 + \left(\frac{\sigma_2^a}{\sigma_2^b} \right)^2 \frac{1 - \exp(-\gamma_2 h)}{\gamma_2 h}}.$$

For small values of the sampling interval h , we have

$$\beta_0 = \frac{(\sigma_2^b)^2 \tilde{\beta}_{0b} + (\sigma_2^a)^2 \tilde{\beta}_{0a}}{(\sigma_2^b)^2 + (\sigma_2^a)^2}, \quad (17)$$

which shows that beta is a linear combination of the permanent and transitory betas. It is interesting to remark that when permanent and transitory betas are equal ($\tilde{\beta}_{0a} = \tilde{\beta}_{0b}$), we have $\beta_0 = \tilde{\beta}_{0b}$ so that the betas are the same at short and large sampling intervals. However, when $\gamma_1 \neq \gamma_2$, the betas need not be the same for all sampling interval so that non-monotonicities are possible.

[Fig. 2](#) presents the graph of β_{0h} as a function of h for selected cases.¹ It shows that if $\tilde{\beta}_{0a} < \tilde{\beta}_{0b}$ (resp. $\tilde{\beta}_{0a} > \tilde{\beta}_{0b}$) then β_{0h} is a strictly increasing (resp. decreasing) function of h for any $\gamma_i > 0$. However, when $\tilde{\beta}_{0a} = \tilde{\beta}_{0b}$, the true value β_{0h} is independent of h if $\gamma_1 = \gamma_2$ and is a non-monotonic function of h if $\gamma_1 \neq \gamma_2$ (the non-monotonicity is due to the fact that when the permanent betas are the same, different mean-reversion parameters have a short-term effect on the betas and as the sampling interval increases they go back to the same value). Note that if γ_i is very small, $\beta_{0h} = \beta_0$ and there is no sampling interval effect.

4.2. Properties of estimates of betas

We now turn to the properties of estimates of β_h . When using log returns, the regression associated with the Capital Asset Pricing Model (CAPM) for any given sampling interval h is given by

$$R_1(th) = \alpha_{0h} + \beta_{0h} R_2(th) + e(th), \quad (18)$$

with

$$\alpha_{0h} = \left(\alpha_1 - \frac{(\sigma_1^b)^2}{2} \right) h^{1/2} - \beta_{0h} \left(\alpha_2 - \frac{(\sigma_2^b)^2}{2} \right) h^{1/2},$$

and

$$e(th) = \eta_1(th) - \beta_{0h} \eta_2(th).$$

The ordinary least-squares estimate of β_h is:

$$\hat{\beta}_h = \frac{\sum_{t=1}^T (R_1(th) - \bar{R}_1)(R_2(th) - \bar{R}_2)}{\sum_{t=1}^T (R_2(th) - \bar{R}_2)^2}, \quad (19)$$

where $\bar{R}_i = T^{-1} \sum_{t=1}^T R_i(th)$ for $i = 1, 2$.

¹ The parameter values are described in [Table 2](#) (see [Section 5.1](#) for more details). For the case $\tilde{\beta}_{0a} = \tilde{\beta}_{0b}$, it corresponds to portfolio P1, when $\tilde{\beta}_{0a} > \tilde{\beta}_{0b}$ to portfolio P2 and when $\tilde{\beta}_{0a} < \tilde{\beta}_{0b}$ to portfolio P4. For each case, the following values of γ_1 and γ_2 are used: $\gamma_1 = \gamma_2 = .2$, $\gamma_1 = .6$ and $\gamma_2 = .2$ (case $\gamma_1 > \gamma_2$) and $\gamma_1 = .2$ and $\gamma_2 = .6$ (case $\gamma_1 < \gamma_2$).

Proposition 4. For any sampling interval h , we have, as $T \rightarrow \infty$:

$$T^{1/2} (\hat{\beta}_h - \beta_{0h}) \rightarrow^d N(\mathbf{0}, V_h), \tag{20}$$

with

$$V_h \equiv \lim_{T \rightarrow \infty} T \text{var} (\hat{\beta}_h - \beta_{0h}) \tag{21}$$

$$= \frac{(\sigma_1^a)^2 \frac{1 - \exp(-\gamma_1 h)}{\gamma_1 h} + (\sigma_1^b)^2 - \beta_{0h}^2 \left[(\sigma_2^a)^2 \frac{1 - \exp(-\gamma_2 h)}{\gamma_2 h} + (\sigma_2^b)^2 \right]}{(\sigma_2^b)^2 + (\sigma_2^a)^2 \frac{1 - \exp(-\gamma_2 h)}{\gamma_2 h}}.$$

If $h \rightarrow 0$, we have:

$$V_0 \equiv \lim_{h \rightarrow 0} V_h = \frac{(\sigma_1^b)^2 + (\sigma_1^a)^2 - \beta_0^2 \left[(\sigma_2^b)^2 + (\sigma_2^a)^2 \right]}{(\sigma_2^b)^2 + (\sigma_2^a)^2},$$

and if $h \rightarrow \infty$,

$$V_\infty \equiv \lim_{h \rightarrow \infty} V_h = \tilde{\beta}_{0b}^2 (1 - \rho_b^2) / \rho_b^2,$$

which corresponds to the asymptotic variance for any fixed h in the absence of a transitory component.

The proof of this result is quite standard and omitted. What it basically says is that the estimated betas will be close to the true betas as defined by β_{0h} . Hence, we can approximate the behavior of the estimated betas by the behavior of the population values as h varies. Of interest also, is the fact that when h is large, the variance of the estimated beta is directly proportional to the permanent beta. This last convergence result for $\hat{\beta}_h$ in conjunction with Proposition 3 concerning the behavior of β_{0h} as a function of h has the following implications:

- In the absence of a transitory component, there is no sampling interval effect on the estimated betas, for large enough sample sizes;
- If the transitory and permanent betas are equal there will, to a first approximation (i.e., with a large enough sample), also be no sampling interval effect if the mean reversion parameter of the transitory component is the same for the asset and the market portfolio;
- With a transitory component in prices and a difference between the permanent and transitory betas, the limit of $\hat{\beta}_h$ when h increases (which corresponds to the permanent beta) can be less than or greater than β_0 (the limit as h goes to zero) which is a linear combination of the permanent and transitory beta. The sign of the difference will depend on the sign of the difference between the permanent and transitory beta.

We can explain the systematic bias $(E\hat{\beta}_h - \beta_0)$ in terms of some relations which have a direct link with the size of a firm. To make explicit these relations, we first define some concepts.

Definition 2. We say that there is under-evaluation of the beta when $\tilde{\beta}_{0b} < \beta_0$. Conversely, we say that there is over-evaluation when $\tilde{\beta}_{0b} > \beta_0$.

These definitions only involve the limiting values β_0 and $\tilde{\beta}_{0b}$. Intuitively, the sampling interval h can be interpreted for an investor as the horizon of the investments' profitability (Levhari and Levy (1977)). Under the hypothesis that the investors often choose a short horizon for such purposes, the beta corresponding to the "true horizon" would be β_0 . In general, β_0 is a linear combination of the permanent and transitory betas. If these are equal, then the beta at a short horizon (β_0) is the same as the beta at a long horizon ($\tilde{\beta}_{0b}$) and increases in the sampling interval involve no biases.

Consider now the case where the transitory beta is smaller than the permanent one ($\tilde{\beta}_{0a} < \tilde{\beta}_{0b}$). This implies that the systematic risk is larger for the long term than over the short term. Hence, following Banz (1981), we may interpret this case as applying to small firms. Thus, for small firms, the limit of $\hat{\beta}_h$ increases if the sampling interval increases and there is over-evaluation of the betas (this follows since, if $\tilde{\beta}_{0b} > \tilde{\beta}_{0a}$, we have, taking the limit as $h \rightarrow 0$, $\tilde{\beta}_{0b} > \beta_h > \beta_0 > \tilde{\beta}_{0a}$).

The case with the permanent beta smaller than the transitory beta is one where short-term considerations account for more of the long-term risk. We may thus expect this case to apply to less risky or larger firms. Thus for large firms, we have the opposite relation, namely $\hat{\beta}_h$ decreases with an increase in the sampling interval and there is under-evaluation of the betas.

Our framework, indeed, helps to provide an alternative interpretation of the relation between estimated betas or systematic risk and the size of the firms. For short horizons, we should expect the co-movement of returns to be roughly similar for small and large firms. At longer horizons, things are quite different for the two groups when transitory components are present. With a

longer horizon, we should expect small firms to be more risky than the market since they have more chances of undergoing big changes (either bankruptcy or a large growth). This translates into our framework in saying that their permanent (or long-horizon) beta is larger than their transitory (or short-term) beta. For large or well-established firms, we can expect that if some short-term movement in return occurs, it is more likely to be smoothed out in the future (for example a negative shock is less likely to send a large firm into bankruptcy and a positive one to make their value double in a year). This translates in our framework in saying that their permanent beta is less than their transitory beta.

5. Simulation experiments

In this section, we verify if the theoretical results obtained provide an adequate description of the finite sample properties of the estimates of the betas and if these are robust to various changes in the parameters. As interesting cases for the simulations, we consider the three cases depicted in Fig. 2, namely:

- Case 1: $\tilde{\beta}_{0b} = \tilde{\beta}_{0a}$. If the permanent and transitory betas are identical, then the limit of $\hat{\beta}_h$ is independent of h when the coefficients γ_i (which control the degree of mean-reversion) are equal ($\gamma_1 = \gamma_2$). However, if $\gamma_1 \neq \gamma_2$, the limit of $\hat{\beta}_h$ is a non-monotonic function of h .
- Case 2: $\tilde{\beta}_{0b} < \tilde{\beta}_{0a}$. If the permanent beta is less than the transitory beta, the limit of $\hat{\beta}_h$ decreases as the sampling interval increases and there is under-evaluation of the betas.
- Case 3: $\tilde{\beta}_{0b} > \tilde{\beta}_{0a}$. If the permanent beta is greater than the transitory beta, $\hat{\beta}_h$ increases with an increase in the sampling interval and there is over-evaluation of the betas.

5.1. Calibration of the model

To calibrate the model, we first start by normalizing β_0 to 1. This leads us to retain values of $\tilde{\beta}_{0a} = \rho_a \sigma_1^a / \sigma_2^a$ and $\tilde{\beta}_{0b} = \rho_b \sigma_1^b / \sigma_2^b$ which satisfy for case 2 the inequality $\tilde{\beta}_{0b} < \tilde{\beta}_{0a}$, and for case 3, the inequality $\tilde{\beta}_{0a} < \tilde{\beta}_{0b}$. For case 1, we have the equality $\tilde{\beta}_{0b} = \tilde{\beta}_{0a} = 1$ when $\gamma_1 = \gamma_2$. We select the values of ρ_a , σ_1^a , σ_2^a , ρ_b , σ_1^b , and σ_2^b to have five base cases, see Table 2. The first, P1, specifies that the asset or portfolio has a permanent and a transitory beta which are equal $\tilde{\beta}_{0b} = \tilde{\beta}_{0a} = 1$. For the second portfolio, P2, the permanent beta is much less than the transitory beta ($\tilde{\beta}_{0b} = 0.15$ and $\tilde{\beta}_{0a} = 1.35$). For the third portfolio, the difference between the transitory and permanent betas is reduced ($\tilde{\beta}_{0b} = 0.90$ and $\tilde{\beta}_{0a} = 1.35$). For portfolios 4 and 5, the specifications are the same as for portfolios 2 and 3 except that we interchange the values for $\tilde{\beta}_{0a}$ and $\tilde{\beta}_{0b}$.

The values retained for the coefficients γ_i are 0.20, 0.60 and 0.01. The value 0.01 is considered to illustrate the effect of a weak reversion to the mean for the transitory component. Here, we can no longer really consider that component as transitory since it is nearly integrated, and we would expect to have results corresponding to the no-transitory component case. The other values are such that they imply autoregressive coefficients of 0.98 and 0.95 selected by Poterba and Summers (1988) with monthly data ($-0.20 \approx 12 \ln(0.98)$ and $-0.6 \approx 12 \ln(0.95)$). Given the absence of any empirical results giving information on the relative magnitude of γ_1 (the mean-reversion coefficient for a stock or portfolio) and γ_2 (the mean-reversion coefficient for the market portfolio), we set $\gamma_1 = \gamma_2$ in the base case. However, given that returns are ARMA(1,1) stationary processes for any fixed h , it is likely that the effect of a shock on the transitory component of prices becomes negligibly faster than for the market portfolio for some types of assets and slower for others. Hence, we also assess the extent to which the results are sensitive to setting $\gamma_2/\gamma_1 < 1$ or $\gamma_2/\gamma_1 > 1$.

The sampling interval h considered are $h = 0, 1, 2, 4, 8, 12, 24$, and ∞ and we set $T = 200$ (other values of T gave similar qualitative results). For a given sampling interval h , we simulate T independent realizations of the processes $u^*(th) = (u_1^*(th), u_2^*(th))'$ and $v^*(th) = (v_1^*(th), v_2^*(th))'$ from a multivariate $N(0, \Omega)$ distribution where Ω is the variance-covariance matrix of the process $u^*(th)$ or $v^*(th)$ (see Proposition 1). We then construct the processes $\eta_i^*(th)$ ($i = 1, 2$) and deduce from them the returns $R_1^*(th)$ and $R_2^*(th)$ and estimate β_h from Eq. (19). We repeat this procedure 3000 times to obtain the mean of the estimator.

Table 2
Selected parameter values.

	Portfolio				
	P1	P2	P3	P4	P5
ρ_a	0.6000	0.7000	0.7000	0.1000	0.6000
σ_1^a	2.0000	3.0055	1.0309	1.5000	1.5000
σ_2^a	1.2000	1.5584	0.5345	1.0000	1.0000
ρ_b	0.7500	0.1000	0.6000	0.7000	0.7000
σ_1^b	1.0000	1.5000	1.5000	3.0055	1.0309
σ_2^b	0.7500	1.0000	1.0000	1.5584	0.5345
$\tilde{\beta}_{0a}$	1.0000	1.3500	1.3500	0.1500	0.9000
$\tilde{\beta}_{0b}$	1.0000	0.1500	0.9000	1.3500	1.3500

Table 3
Mean of the estimated beta.

h	0	1	2	4	8	12	24	∞
$\gamma_1 = \gamma_2 = 0.20.$								
P1	1.003	0.998	1.003	1.001	1.000	0.997	1.001	1.002
P2	1.002	0.974	0.952	0.902	0.809	0.724	0.556	0.150
P3	1.002	0.991	0.989	0.977	0.957	0.941	0.928	0.902
P4	1.003	1.021	1.049	1.089	1.145	1.183	1.258	1.352
P5	1.002	1.005	1.018	1.034	1.065	1.091	1.163	1.352
$\gamma_1 = \gamma_2 = 0.60.$								
P1	1.003	0.999	1.003	1.002	1.000	0.997	1.001	1.002
P2	1.002	0.924	0.854	0.727	0.553	0.452	0.329	0.150
P3	1.002	0.978	0.967	0.947	0.925	0.914	0.911	0.902
P4	1.003	1.064	1.122	1.192	1.255	1.281	1.318	1.352
P5	1.002	1.022	1.050	1.096	1.162	1.199	1.264	1.352
$\gamma_1 = \gamma_2 = 0.01.$								
P1	1.003	0.998	1.003	1.000	1.001	0.998	1.001	1.002
P2	1.002	0.997	0.999	0.995	0.993	0.986	0.975	0.150
P3	1.002	0.998	1.003	1.001	0.998	0.992	0.994	0.902
P4	1.003	0.999	1.006	1.007	1.009	1.010	1.030	1.352
P5	1.002	0.998	1.003	1.002	1.005	1.004	1.011	1.352

5.2. The finite sample mean of β_h in the base case

The results are presented in Table 3 for the cases $\gamma_1 = \gamma_2 = .20$, $\gamma_1 = \gamma_2 = .60$ and $\gamma_1 = \gamma_2 = .01$. In general, the results support the theoretical findings of Section 4. If the transitory and permanent betas are equal (P1), there is indeed no sampling interval effect for any value of the mean-reversion coefficients γ_1 and γ_2 . However, when the transitory beta is greater than the permanent beta, $\tilde{\beta}_{0a} > \tilde{\beta}_{0b}$, the mean of the estimated beta decreases as h increases while the opposite holds when $\tilde{\beta}_{0a} < \tilde{\beta}_{0b}$. This rate of decrease (when $\tilde{\beta}_{0a} > \tilde{\beta}_{0b}$) or increase (when $\tilde{\beta}_{0a} < \tilde{\beta}_{0b}$) is faster the larger is the difference between $\tilde{\beta}_{0a}$ and $\tilde{\beta}_{0b}$. The differences are also more important when the mean-reversion coefficient increases (i.e., from 0.2 to 0.6). When the mean-reversion coefficients are set to 0.01, we see that the bias practically disappears. This is to be expected, since with such small value there is no longer a temporary component since it is almost integrated.

5.3. Sensitivity analyses

To study the sensitivity of the results to changes in various parameters, we consider, as a basis for reference, the case where $\gamma_1 = \gamma_2 = 0.20$ and the difference between $\tilde{\beta}_{0a}$ and $\tilde{\beta}_{0b}$ is large, for example $\tilde{\beta}_{0a} > \tilde{\beta}_{0b}$ with $\tilde{\beta}_{0a} = 1.35$ and $\tilde{\beta}_{0b} = 0.15$ (Table 2 with P2). We performed simulations using different cases as a basis for reference and the conclusions are similar.

The sensitivity of the results is analyzed in three directions in relation to the sub-groups of parameters (γ_1, γ_2) , $(\rho_a, \sigma_1^a, \sigma_2^a)$ and $(\rho_b, \sigma_1^b, \sigma_2^b)$. The strategy is to vary the parameters of one group while keeping the others constant. We first consider variations in the parameters (γ_1, γ_2) and in particular on the effect of specifying $\gamma_1 > \gamma_2$ or $\gamma_1 < \gamma_2$. Secondly, we analyze the effect of changing the parameters of the variance–covariance matrix of the transitory component keeping constant $\tilde{\beta}_{0a} = \rho_a \sigma_1^a / \sigma_2^a$. Finally, we examine the effect of changes in the parameters $(\rho_b, \sigma_1^b, \sigma_2^b)$, related to the permanent component, keeping constant $\tilde{\beta}_{0b} = \rho_b \sigma_1^b / \sigma_2^b$.

Consider first the effect of changes in the parameter γ_i . Table 4 presents the mean of the estimated betas as a function of h for different values of γ_1 (resp. γ_2) when γ_2 (resp. γ_1) is fixed, the reference curve corresponding to the case $\gamma_1 = \gamma_2 = 0.20$. We observe that the monotonically decreasing behavior of the estimated betas is not affected by alternative choices of the mean-reversion parameters.

Consider now the effect of changes in the parameters $(\rho_a, \sigma_1^a, \sigma_2^a)$ and $(\rho_b, \sigma_1^b, \sigma_2^b)$. We analyze jointly the effect of changes in the parameters ρ_a, σ_1^a and σ_2^a (resp. ρ_b, σ_1^b and σ_2^b) keeping $\tilde{\beta}_{0a} = \rho_a \sigma_1^a / \sigma_2^a$ (resp $\tilde{\beta}_{0b} = \rho_b \sigma_1^b / \sigma_2^b$) and the ratio σ_2^a / σ_1^a fixed (the

Table 4
Effect of the mean-reversion coefficients on the mean of the estimated beta.

h	0	1	2	4	8	12	24	∞
$\gamma_1 = 0.01 \quad \gamma_2 = 0.20$	1.002	0.978	0.961	0.921	0.843	0.771	0.630	0.150
$\gamma_1 = 0.60 \quad \gamma_2 = 0.20$	1.002	0.856	0.754	0.617	0.483	0.417	0.332	0.150
$\gamma_1 = 3 \quad \gamma_2 = 0.20$	1.002	0.409	0.277	0.206	0.173	0.163	0.162	0.150
$\gamma_1 = 0.20 \quad \gamma_2 = 0.01$	1.002	0.918	0.849	0.726	0.559	0.450	0.299	0.150
$\gamma_1 = 0.20 \quad \gamma_2 = 0.60$	1.002	0.969	0.936	0.860	0.712	0.599	0.426	0.150
$\gamma_1 = 0.20 \quad \gamma_2 = 3$	1.002	0.738	0.591	0.459	0.348	0.293	0.234	0.150

Table 5

Effect of the coefficient of the variance–covariance matrix on the mean of the estimated beta.

h	0	1	2	4	8	12	24	∞	
Base case: $\gamma_1 = \gamma_2 = 0.20$, $\sigma_2^2/\sigma_1^2 = 1.558$, $(\rho_b, \sigma_1^b, \sigma_2^b) = (0.10, 1.50, 1.00)$, and $\sigma_2^2 = 1.558$.									
$\rho_a = 0.10$	$\sigma_1^a = 21.038$	1.015	0.971	0.949	0.906	0.791	0.716	0.559	0.150
$\rho_a = 0.40$	$\sigma_1^a = 5.2596$	1.005	0.973	0.952	0.902	0.806	0.723	0.557	0.150
Base case: $\gamma_1 = \gamma_2 = 0.20$, $\sigma_2^2/\sigma_1^2 = 1.558$, $(\rho_b, \sigma_1^b, \sigma_2^b) = (0.70, 3.005, 1.558)$, and $\sigma_2^2 = 1.0$.									
$\rho_b = 0.40$	$\sigma_1^b = 0.375$	1.017	0.975	0.952	0.901	0.808	0.723	0.552	0.150
$\rho_b = 0.70$	$\sigma_1^b = 0.214$	1.018	0.975	0.952	0.900	0.808	0.724	0.550	0.150

reference values are those in Table 2 for the case P2). From the results, presented in Table 5, we see that the sampling interval effect remains. Concerning the parameters ρ_b , σ_1^b and σ_2^b , the results (compared to the reference case) show that variations in these parameters do not significantly affect the mean of the estimated betas. However, if σ_2^b is very large relative to σ_1^b , there can be large dispersions in small samples when h is small.

6. Empirical evidence

We have shown that the presence of transitory components in stock returns could generate the empirical patterns in the estimates of the betas computed using different sampling intervals. In order to make our argument more convincing, it is useful to see if there is enough evidence of such transitory components in the data for the various portfolios considered. This is a delicate issue. Our model is (by design) simple to keep the main features of interest and abstract from many others that can affect the stochastic properties of the returns. This is especially the case for short horizons for which it is well documented that returns are positively correlated for small firms due to features such as thin trading. Hence, we certainly do not and cannot claim that our model is a full and adequate description of the data. But we can still try to assess whether mean reversion is indeed present by looking at medium to long-horizons.

6.1. ARMA(1,1) estimates and implied mean reversion

In our model, stock prices have transitory components and the errors of the permanent and transitory components are i.i.d., hence the demeaned returns $r_i(th)$ follow ARMA(1,1) processes. More specifically, demeaned stock returns for portfolio i are

Table 6

MLE estimates and likelihood ratio test results for the ARMA(1,1) model.

Portfolio	$(1 - \phi_1 L)R_i(th) = (1 - \theta_1 L)\varepsilon_i(th)$					
	Horizons of 6 months			Horizons of 12 months		
	$\hat{\phi}$	$\hat{\theta}$	LR test	$\hat{\phi}$	$\hat{\theta}$	LR test
MV1	0.870	0.965	6.51[0.04]	0.744	0.896	3.14[0.21]
MV2	0.858	0.957	6.00[0.05]	0.785	0.943	5.84[0.05]
MV3	0.848	0.942	4.98[0.08]	0.810	0.925	4.30[0.12]
MV4	0.863	0.942	4.69[0.10]	0.800	0.915	4.65[0.10]
MV5	0.876	0.954	5.29[0.07]	0.847	0.943	3.80[0.15]
MV6	0.895	0.957	5.27[0.07]	0.856	0.932	3.76[0.15]
MV7	0.830	0.935	5.41[0.07]	0.724	0.892	4.76[0.09]
MV8	0.901	0.959	4.83[0.09]	0.854	0.930	3.88[0.14]
MV9	0.863	0.969	7.22[0.03]	0.847	0.999	6.48[0.04]
MV10	0.870	0.959	5.47[0.07]	0.828	0.952	4.69[0.10]
MV11	0.821	0.936	4.70[0.10]	0.746	0.905	4.88[0.09]
MV12	0.862	0.953	5.20[0.07]	0.805	0.934	4.62[0.10]
MV13	0.857	0.956	5.50[0.06]	0.749	0.925	5.43[0.07]
MV14	0.877	0.956	4.99[0.08]	0.835	0.941	3.71[0.16]
MV15	0.882	0.960	5.28[0.07]	0.857	0.959	4.06[0.13]
MV16	0.919	0.967	3.87[0.14]	0.935	0.999	2.87[0.24]
MV17	0.881	0.953	3.74[0.15]	0.833	0.936	3.47[0.18]
MV18	0.944	0.999	4.13[0.13]	0.711	0.813	1.12[0.57]
MV19	0.912	0.975	4.31[0.12]	0.908	0.999	4.09[0.13]
MV20	0.943	0.987	2.76[0.25]	0.116	0.000	0.98[0.61]
EW	0.860	0.954	5.98[0.05]	0.804	0.932	4.74[0.09]

Note: Entries in bold indicate significance at least at the 10% level. The p -values are given in parenthesis.

generated as,

$$(1 - \phi_i L)r_i(th) = (1 - \theta_i L)\varepsilon_i(th),$$

where $\phi_i \equiv \exp(-\gamma_i h)$ and the MA parameter given by

$$\theta_i = \frac{(1 + \phi_i^2)(\sigma_i^b)^2 + \frac{1 - \phi_i^2}{\gamma_i h}(\sigma_i^a)^2 - (1 - \phi_i) \left[\frac{2(1 - \phi_i^2)}{\gamma_i h} \left(\frac{\sigma_i^a}{\sigma_i^b} \right)^2 + (1 + \phi_i)^2 \right]^{\frac{1}{2}} (\sigma_i^b)^2}{\frac{1 - \phi_i^2}{\gamma_i h}(\sigma_i^a)^2 + 2\phi_i(\sigma_i^b)^2}. \tag{22}$$

Also, $\varepsilon_i(th)$ is *i.i.d* with variance $\sigma_\varepsilon^2 = (1/\theta_i)((1/2\gamma)\phi_i(1 - \phi_i^2)(\sigma_i^a)^2 + (\sigma_i^b)^2)$. In this section, we fit this ARMA(1,1) model to the demeaned returns of the 20 portfolios measured using sampling intervals of 6 and 12 months and test the null hypothesis of *i.i.d.* returns versus this ARMA(1,1) using a likelihood ratio test. The exact specifications of the estimation procedure and the test are presented in the “computational appendix”.

Table 6 presents the results. The ARMA(1,1) parameter estimates and the test statistics are in bold if they are significant at least at the 10% level. Note first that in all cases (except MV20 at the 12 month interval), the estimate of the MA parameter is larger than that of the AR parameter implying negative serial correlation in returns. We can indeed find significant evidence of mean-reverting behavior in many of the MV portfolios. For the semi-annual returns, we are able to reject the null at the 10% significance level for 16 portfolios, including the market portfolio. The test statistics are significant at the 5% level for the first two smallest portfolios. With annual returns, we can reject the null at the 10% level for 9 portfolios, including the market portfolio. Note that we cannot find any significant evidence of mean-reversion for the five largest MV portfolio returns, which might be due to the presence of a small mean-reversion parameter. The mean-reversion parameter, γ_i , can easily be inferred from our MLE estimates of the autoregressive parameters. For the semi-annual returns, the largest (significant) autoregressive parameter estimate is 0.90 and the smallest is 0.83. Since $\gamma_i = -\ln(\phi_i)/h$, we can infer that the mean-reversion parameters range between 0.2 and 0.37. With annual returns, they range between 0.17 and 0.32. These values of the mean-reversion parameters imply a monthly autoregressive coefficient ranging between 0.97 and 0.98. The following display compares the averages of the mean-reversion parameter estimates among four groups of different MV portfolios.²

Averages of mean-reversion parameters		
Portfolios	Intervals	
	6	12
MV1–MV5	0.30	0.23
MV6–MV10	0.28	0.20
MV11–MV15	0.30	0.23
MV16–MV20	0.17	0.17

The first row reports the average γ_i for the five smallest portfolio returns and the last row reports the average γ_i for the five largest portfolio returns. The results show that the mean-reversion parameters are indeed smaller for the largest MV portfolios but the point estimates remain economically important.

6.2. Long-horizon regressions

In this section, we estimate long-horizon regressions to see if there is evidence of transitory components in stock prices. The long-horizon regressions considered take the form

$$R_i(th) = a_h + \rho(h)R_i((t-1)h) + \varepsilon_i(th), \tag{23}$$

where we consider values of h ranging from 1 year to 10 years. Tables 7a and 7b report the OLS estimates of $\rho(h)$ in panel (a) and the corresponding t -statistic in panel (b). As is well-known, in such regressions the observations are overlapping and the disturbances $\varepsilon_i(th)$ are serially correlated. Therefore, we adjust the standard errors in the t -statistic using a standard heteroskedasticity and autocorrelation consistent (HAC) estimate of the variance based on the Bartlett kernel with the bandwidth selected using the data-dependent procedure recommended by Andrews (1991) and Andrews and Monahan (1992).

² Note: In calculating the average for MV16–MV20 for a 12 month interval, we do not include MV20 since the returns are estimated as a low-order AR(1) process.

Table 7a
Results for the long-horizon regressions; market-value portfolio returns MV1–MV10.

Portfolio	Return horizon										GMM	WAC
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 year	9 years	10 years		
a. OLS estimate of $\rho(h)$												
MV1	-0.02 [0.38]	-0.15 [0.18]	-0.27 [0.07]	-0.46 [0.01]	-0.51 [0.01]	-0.36 [0.10]	-0.22 [0.32]	-0.15 [0.50]	-0.08 [0.64]	-0.04 [0.71]	14.46	20.43 ^a
MV2	0.01 [0.52]	-0.13 [0.22]	-0.35 [0.02]	-0.57 [0.00]	-0.63 [0.00]	-0.51 [0.02]	-0.41 [0.10]	-0.43 [0.13]	-0.37 [0.24]	-0.33 [0.33]	21.23 ^b	21.09 ^a
MV3	0.02 [0.55]	-0.07 [0.37]	-0.28 [0.06]	-0.51 [0.00]	-0.59 [0.00]	-0.47 [0.03]	-0.38 [0.13]	-0.43 [0.11]	-0.46 [0.11]	-0.44 [0.17]	14.80	18.79 ^a
MV4	0.04 [0.60]	-0.08 [0.34]	-0.26 [0.08]	-0.46 [0.01]	-0.51 [0.01]	-0.37 [0.10]	-0.30 [0.23]	-0.37 [0.18]	-0.39 [0.18]	-0.34 [0.31]	15.57	21.59 ^a
MV5	-0.05 [0.33]	-0.15 [0.20]	-0.25 [0.09]	-0.41 [0.02]	-0.45 [0.02]	-0.35 [0.12]	-0.27 [0.26]	-0.34 [0.21]	-0.37 [0.21]	-0.34 [0.31]	19.86 ^b	16.55 ^c
MV6	0.01 [0.50]	-0.08 [0.36]	-0.23 [0.11]	-0.39 [0.02]	-0.43 [0.03]	-0.32 [0.15]	-0.25 [0.29]	-0.32 [0.23]	-0.36 [0.21]	-0.34 [0.29]	17.82 ^c	14.53
MV7	-0.05 [0.32]	-0.17 [0.16]	-0.31 [0.04]	-0.45 [0.01]	-0.49 [0.01]	-0.39 [0.08]	-0.31 [0.21]	-0.34 [0.22]	-0.33 [0.26]	-0.29 [0.37]	15.72	11.55
MV8	0.02 [0.56]	-0.07 [0.40]	-0.20 [0.16]	-0.33 [0.06]	-0.37 [0.07]	-0.27 [0.21]	-0.26 [0.27]	-0.36 [0.18]	-0.39 [0.17]	-0.35 [0.27]	21.69 ^b	18.46 ^a
MV9	-0.03 [0.42]	-0.15 [0.20]	-0.26 [0.09]	-0.42 [0.02]	-0.50 [0.01]	-0.43 [0.05]	-0.36 [0.14]	-0.41 [0.13]	-0.46 [0.10]	-0.46 [0.15]	16.36 ^c	11.50
MV10	-0.03 [0.42]	-0.14 [0.22]	-0.25 [0.09]	-0.35 [0.04]	-0.36 [0.07]	-0.27 [0.22]	-0.22 [0.34]	-0.28 [0.28]	-0.33 [0.24]	-0.34 [0.28]	18.03 ^c	10.88
b. <i>t</i> -statistics of $\rho(h)$												
MV1	-0.13	-0.54	-0.93	-1.22	-2.55	-1.33	-0.80	-0.13	-0.20	-0.02		
MV2	0.12	-0.40	-1.67	-2.83	-3.05	-2.40	-0.94	-0.46	-0.37	-0.13		
MV3	0.13	-0.24	-1.47	-2.55	-2.71	-1.88	-1.12	-0.83	-0.94	-0.14		
MV4	0.32	-0.28	-1.67	-3.15	-2.33	-1.63	-0.97	-0.89	-1.35	-0.23		
MV5	-0.33	-0.49	-1.83	-2.46	-1.85	-1.34	-0.86	-1.14	-1.10	-0.27		
MV6	0.04	-0.28	-1.63	-2.38	-1.79	-1.39	-0.75	-0.96	-1.21	-0.42		
MV7	-0.35	-0.59	-2.28	-2.78	-2.52	-1.57	-0.98	-0.60	-1.03	-0.23		
MV8	0.11	-0.26	-1.25	-1.94	-1.55	-1.18	-0.84	-1.87	-1.68	-0.91		
MV9	-0.20	-0.53	-1.30	-1.93	-2.90	-1.77	-1.17	-0.94	-1.75	-0.56		
MV10	-0.15	-0.48	-1.75	-2.21	-1.59	-1.06	-0.72	-1.50	-1.96	-1.00		

Note: Bootstrapped *p*-values are given in brackets; bold entries indicate significance at the 5% level. a, b, and c denote rejection at the 1%, 5% and 10%, respectively.

We also provide the bootstrapped *p*-values for testing whether the individual estimates $\hat{\rho}(h)$ are significantly less than zero. This is useful since inference based on the standard asymptotic distribution theory may not provide a good approximation in finite samples. As noted by Richardson and Stock (1989), the conventional large-sample approximations may fail to perform well given the small number of effective non-overlapping observations in the long-horizon regressions. Second, there is a finite-sample bias in the OLS estimates of $\hat{\rho}(h)$. It can be shown that the empirical distribution of $\hat{\rho}(h)$ under the null hypothesis that $\rho(h) = 0$ tends to be downward-biased and more so as the return horizon increases (see Kendall (1954), Marriott and Pope (1954), Daniel (2001)). To deal with these issues, we follow the literature such as Goetzmann and Jorion (1993), Kothari and Shanken (1997) and Killian (1999) and construct *p*-values of the estimates using a bootstrap method. We first use the stationary bootstrap method of Politis and Romano (1994) to randomly draw (with replacement) a new sample of monthly returns $\{R_i^*(th); i = 1, \dots, T\}$, where $R_i^*(th)$ are drawn in blocks whose starting indices and lengths are determined randomly to preserve the time-series dependence in returns. The block length is drawn from a geometric distribution with a parameter *q* set to 0.1 and the number of bootstrap replications is 5000. The parameter *q* determines the average block length as $b = 1/q$. The results are similar when we set $b = 25$. The distribution of $(\hat{\rho}(h) - \rho(h))$ can then be approximated by the empirical distribution of $(\hat{\rho}^*(h) - \hat{\rho}(h))$, where $\hat{\rho}^*(h)$ is calculated from regressing each bootstrap sample $\{R_i^*(th); i = 1, \dots, T\}$ in the long-horizon regression. By imposing the null hypothesis $\rho(h) = 0$, we can test $H_0: \rho(h) = 0$ against $H_A: \rho(h) < 0$ and compute the *p*-value as the proportion of draws of $(\hat{\rho}^*(h) - \hat{\rho}(h))$ that are less than $\hat{\rho}(h)$. In addition, we studentize the test statistics by dividing $(\hat{\rho}^*(h) - \hat{\rho}(h))$ by the standard deviation of $\hat{\rho}^*(h)$ as advocated by Romano and Wolf (2005) to improve both size and power.

Finally, we evaluate the statistical significance of mean reversion for the MV portfolio returns by jointly testing whether the long-horizon regression coefficients are equal to zero. As pointed out by Richardson (1993), it is better to evaluate the statistical significance of mean reversion using a joint test. We consider the χ^2 joint test based on the GMM framework of Richardson and Smith (1991) and the χ^2 joint test of a set of weighted autocorrelation test statistics of Daniel (2001). The details are laid out in the “computational appendix”.

The results are presented in Tables 7a and 7b. When assessing the statistical significance of a single estimate, most of the HAC *t*-statistics are significant for regressions constructed with 3–5 year return horizons. For the small to mid-size MV portfolio returns, the *t*-statistics are significant at the 4–5 years return horizons, while the statistics are significant at the 3–4 year return

Table 7b
Results for the long-horizon regressions; market-value portfolio returns MV11–MV20.

Portfolio	Return Horizon										GMM	WAC
	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 year	9 years	10 years		
<i>a. OLS estimate of $\rho(h)$</i>												
MV11	−0.07 [0.28]	−0.20 [0.11]	−0.32 [0.04]	−0.39 [0.02]	−0.38 [0.06]	−0.26 [0.23]	−0.22 [0.34]	−0.29 [0.27]	−0.32 [0.27]	−0.32 [0.32]	21.31 ^b	9.66
MV12	−0.04 [0.34]	−0.19 [0.12]	−0.32 [0.03]	−0.38 [0.03]	−0.39 [0.05]	−0.28 [0.20]	−0.22 [0.34]	−0.26 [0.31]	−0.28 [0.32]	−0.28 [0.38]	23.71 ^a	12.45
MV13	−0.02 [0.42]	−0.21 [0.10]	−0.31 [0.05]	−0.31 [0.08]	−0.30 [0.13]	−0.21 [0.32]	−0.18 [0.43]	−0.28 [0.27]	−0.35 [0.21]	−0.36 [0.25]	23.62 ^a	10.54
MV14	−0.05 [0.35]	−0.18 [0.14]	−0.32 [0.03]	−0.36 [0.04]	−0.33 [0.09]	−0.18 [0.36]	−0.09 [0.56]	−0.16 [0.47]	−0.21 [0.42]	−0.25 [0.39]	24.64 ^a	11.26
MV15	−0.04 [0.41]	−0.15 [0.21]	−0.27 [0.08]	−0.26 [0.15]	−0.22 [0.25]	−0.08 [0.58]	0.00 [0.72]	−0.09 [0.60]	−0.17 [0.50]	−0.21 [0.46]	29.01 ^a	10.71
MV16	−0.04 [0.37]	−0.19 [0.13]	−0.24 [0.11]	−0.22 [0.18]	−0.19 [0.30]	−0.05 [0.61]	0.03 [0.77]	−0.04 [0.68]	−0.10 [0.61]	−0.14 [0.57]	27.61 ^a	8.46
MV17	−0.03 [0.44]	−0.22 [0.09]	−0.30 [0.04]	−0.24 [0.15]	−0.18 [0.32]	−0.01 [0.68]	0.08 [0.82]	−0.03 [0.68]	−0.11 [0.57]	−0.17 [0.51]	29.00 ^a	12.62
MV18	0.00 [0.50]	−0.17 [0.16]	−0.33 [0.03]	−0.31 [0.08]	−0.20 [0.28]	−0.01 [0.69]	0.07 [0.81]	−0.03 [0.69]	−0.11 [0.58]	−0.19 [0.48]	33.05 ^a	16.46 ^c
MV19	−0.06 [0.32]	−0.19 [0.12]	−0.29 [0.06]	−0.23 [0.18]	−0.15 [0.39]	0.03 [0.76]	0.13 [0.88]	0.06 [0.81]	−0.01 [0.74]	−0.09 [0.65]	27.32 ^a	10.94
MV20	0.01 [0.53]	−0.12 [0.28]	−0.24 [0.11]	−0.19 [0.25]	−0.10 [0.48]	0.06 [0.79]	0.14 [0.88]	0.10 [0.84]	0.02 [0.77]	−0.07 [0.67]	27.66 ^a	14.92
EW	−0.04 [0.35]	−0.18 [0.15]	−0.33 [0.03]	−0.45 [0.01]	−0.47 [0.01]	−0.34 [0.13]	−0.26 [0.28]	−0.33 [0.22]	−0.36 [0.21]	−0.34 [0.30]	23.44 ^a	14.83
<i>b. t-statistics of $\rho(h)$</i>												
MV11	−0.35	−0.75	−2.44	−2.96	−1.84	−1.10	−0.61	−1.24	−1.23	−0.49		
MV12	−0.20	−0.66	−2.27	−2.58	−2.00	−1.22	−0.75	−1.22	−1.50	−0.58		
MV13	−0.12	−0.63	−2.39	−2.27	−1.52	−1.02	−0.58	−1.78	−2.34	−1.04		
MV14	−0.25	−0.55	−2.70	−2.61	−1.52	−0.82	−0.33	−0.76	−1.21	−0.89		
MV15	−0.16	−0.42	−1.74	−1.70	−1.14	−0.46	−0.01	−0.50	−1.03	−0.98		
MV16	−0.17	−0.57	−1.62	−1.51	−1.08	−0.32	0.15	−0.22	−0.68	−0.90		
MV17	−0.09	−0.61	−2.12	−1.95	−0.98	−0.06	0.40	−0.20	−0.94	−0.85		
MV18	−0.01	−0.43	−1.76	−1.99	−1.06	−0.07	0.30	−0.18	−0.81	−1.18		
MV19	−0.23	−0.45	−1.80	−1.51	−0.96	0.28	0.76	0.41	−0.06	−0.61		
MV20	0.04	−0.26	−0.76	−1.12	−0.61	0.55	0.81	0.27	0.04	−0.06		
EW	−0.28	−0.61	−2.42	−2.94	−2.36	−1.35	−0.73	−0.91	−1.15	−0.36		

Note: Bootstrapped p -values are given in brackets; bold entries indicate significance at the 5% level. a, b, and c denote rejection at the 1%, 5% and 10%, respectively.

horizons for the mid-to-large MV portfolio returns. Similar to the results obtained from the estimation of the ARMA(1,1) model, nine out of the first ten small MV portfolios have at least one significant t -statistic but four out of the six largest MV portfolios do not have significant t -statistics at any horizon. The bootstrapped p -values (given in brackets) imply similar results. Sixteen portfolios including the market portfolio have at least one significant negative autocorrelation at the 4 year horizon and four of the six largest MV portfolios do not have significant negative autocorrelation estimates while only one mid-size MV portfolio does not have a significant estimate.

Of more interest are the results of the joint tests reported in the last two columns, denoted “GMM” for the joint tests of Richardson and Smith (1991) and “WAC” for the joint test of Daniel (2001). When testing whether all the first-order autocorrelation coefficients are zero using one to ten year returns, we can reject the null hypothesis of no mean reversion using the test of Richardson and Smith (1991) for 17 portfolios at the 10% level and for 14 portfolios at the 5% level, including the market portfolio. Using the test of Daniel (2001), we can reject the null for 7 portfolios at the 10% level and 5 portfolios at the 1% level. The test statistics of Richardson and Smith (1991) tend to be more significant for the large MV portfolios while those of Daniel (2001) tend to be more significant for the small MV portfolios. Overall, we view the results as convincing evidence of mean-reversion and the presence of transitory components in stock prices, in accordance with the assumptions of our model.

7. Discussion in relation to previous literature

Of particular interest in relation to our work is that of Lo and MacKinlay (1990). Using weekly stock returns they document the fact that portfolio returns are positively autocorrelated despite the weakly negative autocorrelations of individual returns. They attribute this result to the fact that individual securities are positively (and asymmetrically) cross-autocorrelated and, in particular, that the returns of large stocks lead those of small stocks. They show that even if there is no stock market overreaction

and no negative serial correlation in stock returns, it can be profitable to construct a contrarian investment strategy which is to buy the securities that have performed poorly in the past and sell those that have performed well. They argue that nonsynchronous trading cannot explain the positive autocorrelation and lead-lag effects without requiring unrealistically thin markets. Their finding about the lead-lag effects can imply that small firms react to market news with a delay, so that indeed they have low betas for short periods but with full adjustment occurring the betas can rise over longer periods. So for small firms, this is an alternative explanation to ours. One problem with their model is that the cross-autocovariances are always positive so that they cannot explain why the betas of the large-firm portfolios can decrease as the sampling interval increases. On the contrary, our continuous-time model can explain the decreasing patterns of the betas for the large-firm portfolios; all that is needed are a) the presence of transitory components in the market portfolio returns, which we have documented, and b) the fact that, for large firms, the permanent beta is smaller than the transitory beta, which is highly plausible.

Also of interest is the fact that, with weekly returns, [Boudoukh et al. \(1994\)](#) show that the cross-autocorrelation patterns between the small and the large MV portfolio returns may simply arise as the product of the portfolios' own autocorrelation patterns and the high contemporaneous correlations across portfolios. These findings suggest that the lead-lag effects may not result from the delayed response of small firms to market news but actually from the presence of transitory components. Along the same lines, [Hameed \(1997\)](#), using linear factor models, shows that the lead-lag patterns across different size portfolio returns can be attributable to the differences in the level of time variation in expected stock returns. If the degree of predictable variations in the factor sensitivities or factor loadings differs across the portfolios, asymmetric cross-autocorrelations across different size-portfolios will occur. When this is combined with high contemporaneous correlations across the expected returns of different portfolios, these findings suggest that the lead-lag patterns between small and large firms can again be attributable to the presence of transitory components.

[Chordia and Swaminathan \(2000\)](#) also suggest that trading volume is a significant determinant of the lead-lag pattern in stock returns once firm size is controlled for. [Hou \(2007\)](#) claims that the lead-lag effect between large firms and small firms is more likely an intra-industry effect and once controlled for the lead-lag effect becomes insignificant. Both studies assert that the lead-lag effect arises primarily because certain firms react more sluggishly to common information than do others rather than due to nonsynchronous trading or time-varying expected returns.

In summary, given the evidence presented, we believe that our simple model with transitory components is better able to explain the full pattern of the estimates of the betas using various sampling frequencies. As evidenced by the literature cited above that followed the work of [Lo and MacKinlay \(1990\)](#) it is also consistent with the lead-lag pattern and, moreover, able to explain not only the pattern of the estimates of the betas for small firms but for large firms as well.

8. Conclusion

In this study, we have provided a theoretical framework to analyze the empirically supported effect of the sampling interval used to compute returns on the estimated betas. The model used specifies the presence of both permanent and transitory components in prices as in [Fama and French \(1988\)](#) and [Poterba and Summers \(1988\)](#). As in these papers, the discrete-time representation of returns is an $ARMA(1,1)$ process with negative serial correlation. We have derived the corresponding theoretical value of the beta not only as a function of the sampling interval but also of the various parameters of the permanent and transitory components.

Our theoretical results show the importance of the presence of a transitory component in explaining the effect of the sampling interval. Without such a component the betas and their estimates show no relation to the sampling interval. With it, there is a clear monotonic relation whose sign depends on the difference between what we call the permanent and transitory betas. We argue that small firms which are more risky have a transitory beta smaller than the permanent beta and that this implies a beta which increases as the sampling interval increases. The inverse relation holds for large firms whose transitory beta is greater than the permanent beta. Our theoretical results which rely on asymptotic arguments are shown to yield adequate approximations in finite samples using simulation experiments. The extent to which the presence of a transitory component affects the strength of the sampling interval effect depends on parameters such as the coefficients of mean-reversion for the stock (or portfolio) and for the market portfolio.

What we have established is that, in our framework, it is possible to explain the empirical results about the presence of a sampling interval effect on estimated betas using continuously compounded returns only when a transitory component is present in prices. We interpret this as evidence giving support to the claims made by [Poterba and Summers \(1988\)](#) and [Fama and French \(1988\)](#) about the presence of transitory components in stock prices. Finally, our explanation is not at odds with an alternative one due to [Lo and MacKinlay \(1990\)](#) and has the advantage of being able to explain the pattern of the estimates of the betas for both small and large firms.

Appendix A. Mathematical Appendix

Proof of Proposition 2. From the definition of $\eta_i(th)$ as:

$$\eta_i(th) = v_i(th) + (1 - \exp(-\gamma_i h L))^{-1} (1 - L) u_i(th),$$

we can write, after some manipulations:

$$\eta_i(th) = v_i(th) + u_i(th) + \left(1 - \frac{1}{\phi_i}\right) \sum_{j=1}^{\infty} \phi_i^j u_i((t-j)h),$$

with $\phi_i = \exp(-\gamma_i h)$. Consider first the variance of $R_i(th)$. We have:

$$\begin{aligned} \text{var}(R_i(th)) &= \sigma_{v_i}^2 + \sigma_{u_i}^2 + \left(1 - \frac{1}{\phi_i}\right)^2 \sigma_{u_i}^2 \sum_{j=1}^{\infty} \phi_i^{2j} \\ &= \sigma_{v_i}^2 + \sigma_{u_i}^2 + \sigma_{u_i}^2 \left(1 - \frac{1}{\phi_i}\right)^2 \left(\frac{1}{1 - \phi_i^2} - 1\right) \\ &= \sigma_{v_i}^2 + \frac{2}{1 + \phi_i} \sigma_{u_i}^2. \end{aligned}$$

Substituting for ϕ , $\sigma_{v_i}^2$ and $\sigma_{u_i}^2$, we obtain:

$$\text{var}(R_i(th)) = (\sigma_i^a)^2 \frac{1 - \exp(-\gamma_i h)}{\gamma_i} + (\sigma_i^b)^2 h.$$

Consider now the covariance of $R_i(th)$ and $R_i((t-1)h)$. We have:

$$\begin{aligned} \text{cov}(R_i(th), R_i((t-1)h)) &= \text{cov}(\eta_i(th), \eta_i((t-1)h)) \\ &= -\frac{1 - \phi_i}{\phi_i} \text{cov}\left(u_i((t-1)h), \sum_{j=1}^{\infty} \phi_i^j u_i((t-j)h)\right) \\ &\quad + \left(\frac{1 - \phi_i}{\phi_i}\right)^2 \sum_{k=2}^{\infty} \phi_i^{k-1} \text{cov}\left(u_i((t-k)h), \sum_{j=1}^{\infty} \phi_i^j u_i((t-j)h)\right). \end{aligned}$$

Upon some developments, we obtain:

$$\begin{aligned} \text{cov}(R_i(th), R_i((t-1)h)) &= -\frac{1 - \phi_i}{\phi_i} \phi_i \sigma_{u_i}^2 + \frac{(1 - \phi_i)^2}{\phi_i^3} \sigma_{u_i}^2 \sum_{k=2}^{\infty} \phi_i^{2k} \\ &= -\frac{1 - \phi_i}{1 + \phi_i} \sigma_{u_i}^2 \\ &= -\frac{(1 - \phi_i)^2}{2\gamma_i} (\sigma_i^a)^2 \\ &= -\frac{(1 - \exp(-\gamma_i h))^2}{2\gamma_i} (\sigma_i^a)^2. \end{aligned}$$

We deduce that:

$$\lim_{h \rightarrow 0} \text{cov}(R_i(th), R_i((t-1)h)) = 0,$$

and

$$\lim_{\gamma_i \rightarrow 0} \text{cov}(R_i(th), R_i((t-1)h)) = 0.$$

Consider now the time series representation of R_{th} . To deduce that it is an ARMA(1,1), it suffices to show that $\eta_{th} = (\eta_1(th), \eta_2(th))'$ is an ARMA(1,1) for h fixed. Let

$$\zeta_{th} = \begin{bmatrix} 1 - \exp(-\gamma_1 h)L & 0 \\ 0 & 1 - \exp(-\gamma_2 h)L \end{bmatrix} \begin{bmatrix} \eta_1(th) \\ \eta_2(th) \end{bmatrix},$$

and define:

$$\begin{aligned}\phi_i &= \exp(-\gamma_i h), \\ \sigma_{v_i}^2 &= (\sigma_i^b)^2 h, \\ \sigma_{u_i}^2 &= (\sigma_i^a)^2 \frac{1 - \exp(-2\gamma_1 h)}{2\gamma_1}, \\ \rho_{12}^b &= \rho_b \sigma_1^b \sigma_2^b h, \\ \rho_{12}^a &= \rho_a \sigma_1^a \sigma_2^a \frac{1 - \exp(-(\gamma_1 + \gamma_2)h)}{\gamma_1 + \gamma_2}.\end{aligned}$$

It is easily shown that:

$$\begin{aligned}E\zeta_{th}\zeta'_{th} &= \begin{bmatrix} (1 + \phi_1^2)\sigma_{v_1}^2 + 2\sigma_{u_1}^2 & (1 + \phi_1\phi_2)\rho_{12}^b + 2\rho_{12}^a \\ (1 + \phi_1\phi_2)\rho_{12}^b + 2\rho_{12}^a & (1 + \phi_2^2)\sigma_{v_2}^2 + 2\sigma_{u_2}^2 \end{bmatrix}, \\ E\zeta_{th}\zeta'_{(t-1)h} &= \begin{bmatrix} -\phi_1\sigma_{v_1}^2 - \sigma_{u_1}^2 & -\phi_1\rho_{12}^b - \rho_{12}^a \\ -\phi_2\rho_{12}^b - \rho_{12}^a & -\phi_2\sigma_{v_2}^2 - \sigma_{u_2}^2 \end{bmatrix},\end{aligned}$$

and

$$E\zeta_{th}\zeta'_{(t-k)h} = \mathbf{0},$$

for $k \geq 2$. We conclude that ζ_{th} is a $MA(1)$ process. Accordingly, η_{th} is an $ARMA(1,1)$ for any fixed h . This is also true for $\eta_{th}^* = \eta_{th}/h^{1/2}$ and we have:

$$E(\eta_{th}^*(th))^2 = (\sigma_i^a)^2 \frac{1 - \exp(-\gamma_i h)}{\gamma_i h} + (\sigma_i^b)^2,$$

and

$$E\eta_{1h}^*(th)\eta_{2h}^*(th) = \rho_a \sigma_1^a \sigma_2^a \frac{2 - \exp(-\gamma_1 h) - \exp(-\gamma_2 h)}{(\gamma_1 + \gamma_2)h} + \rho_b \sigma_1^b \sigma_2^b.$$

When $h \rightarrow 0$, η_{th}^* is an *i.i.d.* process such that:

$$\begin{aligned}\eta^*(th) &= v^*(th) + u^*(th) \\ &\sim i.i.d. N\left(\mathbf{0}, \begin{bmatrix} (\sigma_1^b)^2 + (\sigma_1^a)^2 & \rho_a \sigma_1^a \sigma_2^a + \rho_b \sigma_1^b \sigma_2^b \\ \rho_a \sigma_1^a \sigma_2^a + \rho_b \sigma_1^b \sigma_2^b & (\sigma_2^b)^2 + (\sigma_2^a)^2 \end{bmatrix}\right).\end{aligned}$$

Proof of Proposition 3. From the preceding proof, we readily obtain:

$$\begin{aligned}\beta_{0h} &= \frac{\text{cov}(R_1(th), R_2(th))}{\text{var}(R_2(th))} \\ &= \frac{\text{cov}(\eta_1(th), \eta_2(th))}{\text{var}(\eta_2(th))} \\ &= \frac{\rho_a \sigma_1^a \sigma_2^a \frac{2 - \exp(-\gamma_1 h) - \exp(-\gamma_2 h)}{(\gamma_1 + \gamma_2)h} + \rho_b \sigma_1^b \sigma_2^b}{(\sigma_2^a)^2 \frac{1 - \exp(-\gamma_2 h)}{\gamma_2 h} + (\sigma_2^b)^2}.\end{aligned}$$

The limiting values when h converges to 0 or ∞ are easily deduced.

Appendix B. Computational Appendix

Estimation method for the results in Table 6: We estimate an $ARMA(1,1)$ model to the demeaned returns using the maximum likelihood method. As recommended by Box and Jenkins (1976), we set the initial values as $\varepsilon_i(1 \cdot h) = 0$ and $r_i(1 \cdot h)$ to the

observed value and obtain the approximation to the conditional likelihood function for $r_i(th)$, $t=2, \dots, T$ as

$$\begin{aligned} \xi(\phi_i, \theta_i) &= \log f(r_i(T \cdot h), \dots, r_i(2 \cdot h) | r_i(1 \cdot h), \varepsilon_i(1 \cdot h) = 0) \\ &= -\frac{(T-1)}{2} \log(2\pi) - \frac{(T-1)}{2} \log \sigma_\varepsilon^2 - \sum_{t=2}^T \frac{\varepsilon_i(th)^2}{2\sigma_\varepsilon^2}, \end{aligned} \tag{A.1}$$

where $\varepsilon_i(th) = r_i(th) - \phi_i r_i((t-1)h) - \theta_i \varepsilon_i((t-1)h)$. The conditional maximum likelihood estimate of σ_ε^2 is $\hat{\sigma}_\varepsilon^2 = (T-1)^{-1} \sum_{t=2}^T (r_i(th) - \hat{\phi}_i r_i((t-1)h) - \hat{\theta}_i \varepsilon_i((t-1)h))^2$. In order to estimate ϕ_i and θ_i , we use the grid search method to numerically find the values of $\hat{\phi}_i$ and $\hat{\theta}_i$ that maximize (A.1). Since our model restricts $\phi_i, \theta_i \in [0, 1]$, we search for the pair of (ϕ_i, θ_i) that maximizes the log-likelihood function (A.1) for each parameter value between 0 and 0.999 in increments of 0.0005. We test hypotheses about these parameters by the likelihood ratio test. Under the null hypothesis of no transitory component, we have the restriction $\phi_i = \theta_i = 0$ and we can construct the likelihood ratio test as $2[\xi(\hat{\phi}_i, \hat{\theta}_i) - \xi(0, 0)]$ that follows a chi-squared distribution 2 degrees of freedom under the null hypothesis.

The joint tests for mean reversion reported in Tables 7a and 7b: Richardson and Smith (1991) proposed a joint testing procedure based on Hansen's (1982) GMM framework for the long-horizon regressions with overlapping observations. Based on the moment restrictions $E[\varepsilon_i(th)] = 0$ and $E[\varepsilon_i(th)R_i(th)] = 0$, the sampling moment restrictions are

$$g_T(\alpha_h, \rho(h)) = \frac{1}{T} \sum_{t=1}^T \begin{pmatrix} R_i(th) - a_h - \rho(h)R_i((t-1)h) \\ [R_i(th) - a_h - \rho(h)R_i((t-1)h)]R_i(th) \end{pmatrix},$$

and the GMM estimator of $(\alpha_h, \rho(h))$ can be obtained by minimizing

$$g_T(\alpha_h, \rho(h))' W_T g_T(\alpha_h, \rho(h))$$

where W_T is some weighting matrix. The optimal weight matrix of W_T is the inverse of the variance-covariance matrix of $g_T(\alpha_h, \rho(h))$. In jointly testing $H_0: \rho(h) = 0$ for $h = j, \dots, k$, we use the following set of just-identified moment restrictions

$$g_T(\theta) = \frac{1}{T} \sum_{t=1}^T \begin{pmatrix} [r_i(tj) - \rho(j)r_i((t-1)j)]r_i(tj) \\ \vdots \\ [r_i(tk) - \rho(k)r_i((t-1)k)]r_i(tk) \end{pmatrix}, \tag{A.2}$$

where the returns $r_i(th)$ are demeaned. The joint hypothesis that $\rho(h) = 0$ for $h = j, \dots, k$ is tested using the Wald statistic $J = T\hat{\gamma}'[V(\hat{\gamma})]^{-1}\hat{\gamma}$, where $\hat{\gamma}$ is the M -vector of autoregressive parameter estimators, $(\hat{\rho}(j), \dots, \hat{\rho}(k))'$. Richardson and Smith (1991) show that under the null hypothesis the variance-covariance matrix of $V(\hat{\gamma})$ has a representation independent of the data given by

$$V(\hat{\gamma}) = \begin{pmatrix} \frac{2j^2 + 1}{3j} & \dots & \frac{j^2 + s(j, k)}{jk} \\ \vdots & \ddots & \vdots \\ \frac{j^2 + s(j, k)}{jk} & \dots & \frac{2k^2 + 1}{3k} \end{pmatrix}$$

where $s(j, k) = 2 \sum_{l=1}^{\min(j, k)} (j-l) \min(j, k-l)$. The statistic J has a chi-square asymptotic distribution with M degrees of freedom under the null hypothesis.

Daniel (2001) proposed a joint test based on a set of weighted sums of autocorrelations, which is asymptotically equivalent to the test of Richardson and Smith (1991). Imposing $\rho(h) = 0$ for $h = j, \dots, k$ in the moment restrictions (A.2) under the null hypothesis, we have

$$g_T(\theta) = \frac{1}{T} \sum_{t=1}^T \begin{pmatrix} r_i(tj)r_i((t-1)j) \\ \vdots \\ r_i(tk)r_i((t-1)k) \end{pmatrix} = \frac{1}{T} \sum_{t=1}^T \begin{pmatrix} \sum_{s=0}^{2j} \min(s, 2j-s) r_{i,t} r_{i,t+s} \\ \vdots \\ \sum_{s=0}^{2k} \min(s, 2k-s) r_{i,t} r_{i,t+s} \end{pmatrix},$$

where $r_{i,t}$ denotes the monthly returns. Dividing the moment conditions by the single period variance estimator, $\hat{\sigma}^2 = T^{-1} \sum_{t=1}^T r_{i,t}^2$, Daniel (2001) proposed to construct the joint Wald test statistic given by

$$J' = T g_T' \frac{1}{\hat{\sigma}^2} (S_0)^{-1} \frac{1}{\hat{\sigma}^2} g_T = T g_T^* (S_0)^{-1} g_T^*,$$

where S_0 is the variance–covariance matrix whose (p, q) th element is

$$E \left[\left(\sum_{l=0}^{2p} \min(l, 2p-l) \frac{r_{i,t} r_{i,t+l}}{r_{i,t}^2} \right) \left(\sum_{m=0}^{2q} \min(m, 2q-m) \frac{r_{i,t} r_{i,t+m}}{r_{i,t}^2} \right) \right].$$

Using a vector notation, g_T^* can be written as

$$g_T^* = \frac{1}{T} \sum_{t=1}^T \begin{pmatrix} \sum_{s=0}^{2j} \min(s, 2j-s) \frac{r_{i,t} r_{i,t+s}}{r_{i,t}^2} \\ \vdots \\ \sum_{s=0}^{2k} \min(s, 2k-s) \frac{r_{i,t} r_{i,t+s}}{r_{i,t}^2} \end{pmatrix} = \left(\sum_{s=0}^{2j} \min(s, 2j-s) \hat{\rho}_s ; \sum_{s=0}^{2k} \min(s, 2k-s) \hat{\rho}_s \right) = \left(w(j)' \hat{\rho} ; w(k)' \hat{\rho} \right),$$

where $w(j)$ is a k -vector whose i th element is $\max(0, \min(s, 2k-s))$ and $\hat{\rho}$ is a k -vector whose i th element is the sample autocorrelation at lag i . Since $\hat{\rho}$ is distributed spherically under the null hypothesis, $E[\hat{\rho} \hat{\rho}'] = T^{-1}I$, where I is the $k \times k$ identity matrix, and S_0 can be expressed as

$$S_0 = TE[g_T^* g_T^{*'}] = \begin{pmatrix} w(j)' w(j) & \cdots & w(j)' w(k) \\ \vdots & \ddots & \vdots \\ w(k)' w(j) & \cdots & w(k)' w(k) \end{pmatrix},$$

The statistic J' has an asymptotic chi-square distribution with M degree of freedom under the null hypothesis.

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